Convex skeleton: Generalization of a network spanning tree

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Convexity is a property of a part of a mathematical object that includes all the shortest paths between its units. In the case of graphs or networks, a connected induced subgraph is said to be convex if every shortest path between the nodes of the subgraph lies entirely within the subgraph [3]. A convex network can be defined as a network such that every connected induced subgraph is convex [4]. Fully convex network would therefore have to be a collection of cliques stitched together in a tree.

We study the largest high-convexity part of empirical networks obtained by removing the least number of edges, which we call a convex skeleton [5]. A convex skeleton is a generalization of a network spanning tree in which each edge can be replaced by a clique of arbitrary size. We present different approaches for extracting convex skeletons and apply them to social collaboration and protein interactions networks, autonomous systems graphs and food webs.

We show that the extracted convex skeletons retain the network degree distribution, clustering, connectivity and also distances between the nodes, while making the shortest paths between the nodes largely unique. Furthermore, convex skeletons retain the network community structure and relative importance of the nodes measured by different node centrality measures. This is in contrast to some popular network backboning techniques such as a spanning tree or high-betweenness backbone [1] and high-salience skeleton [2]. At the same time, a convex skeleton implies a very plain structure consisting only of cliques and a tree.

A convex skeleton thus represents a simple definition of a network backbone. It seems to provide a particularly reasonable abstraction of social collaboration networks such as co-authorship networks, with applications in network modeling, visualization, navigation and possibly also elsewhere.

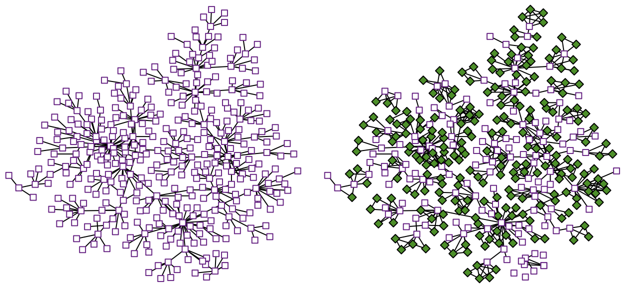

Figure 1: (*left*) A spanning tree and (*right*) a convex skeleton of the network scientists co-authorship network. The nodes with the clustering coefficient above $C > 0.5$ are represented with ◆ and the remaining nodes with □.

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[3] F. Harary and J. Nieminen. Convexity in graphs. *J. Differential Geom.*, 16(2):185–190, 1981.

[4] T. Marc and L. Šubelj. Convexity in complex networks. *Netw. Sci.*, pages 1–28, 2018.

[5] L. Šubelj. Convex skeletons of complex networks. *sub. to J. R. Soc. Interface*, pages 1–19, 2017.

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